International Journal of Mathematics, Statistics and Operations Research
Vol. 2; Number 2; 2022, pp. 289-294
https://doi.org/10.47509/IJMSOR.2022.v02i02.12

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Academic Open Access Publishing
www.arfjournals.com

OPTIMIZING MULTI-OBJECTIVE FUZZY ECONOMIC ORDER QUANTITY MODEL FOR MULTI DETERIORATING ITEMS

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Abstract:

This paper develops a multi-objective, multi-item inventory model for deteriorating items with price-dependent demand under limited storage area and total investment in a fuzzy environment. The objectives are to maximize profit and minimize wastage cost, with profit goal, wastage cost, storage area, and total investment treated as fuzzy parameters. Linear membership functions are used to represent fuzziness, and the model is solved using a fuzzy programming technique. Results through numerical illustration demonstrate the effectiveness of the proposed approach.

Keywords: Multi-objective, Fuzzy, Multi-item, Deteriorating Items

1. Introduction:

In multi-objective mathematical programming problems, a decision maker is required to optimize two or more objectives simultaneously over a given set of decision variables. Generally, a compromise solution is selected among a set of possible solutions. So many methods such as weighting method, assigning priorities to objectives, setting aspiration levels for objectives etc. are available to obtain compromise solution.

In many inventory models objectives, constraints and inventory parameters such as inventory costs, demand, time horizon, deterioration rate etc. are taken as constant. But in real life situations these values may be imprecise in nature. Hence in several inventory problems the objectives and the constraints are described with vagueness and uncertainty in non-stochastic sense and the values of the parameters have some ambiguity. Few years back such kind of uncertainty was outside of the preview of consideration due to the lack of appropriate mathematical tools. But now days it can be effectively modeled using fuzzy set theory. The fuzzy set theory was developed for a domain in which description of activities and observations are fuzzy in the sense that there are no well-defined boundaries of the set of activities or observations to which the description is applied. Fuzzy set theory was initiated by Zadeh [5] and later applied to different real-life situations by several researchers. Zadeh showed the

Article History
Received : 31 August 2022; Revised : 09 October 2022; Accepted : 26 November 2022; Published : 29 December 2022

To cite this paper
H.P. Umap (2022). Optimizing Multi-Objective Fuzzy Economic order Quantity Model for Multi Deteriorating Items. *International Journal of Mathematics, Statistics and Operations Research.* 2(2), 289-294.

intention to accommodate uncertainty in the non-stochastic sense rather than the presence of random variables. Bellman and Zadeh [5] first applied fuzzy set theory in decision making process. Zimmerman [6] used the concept of fuzzy set in decision making process by considering the objectives and constraints as fuzzy goals. He first applied fuzzy set theory with suitable choice of membership functions and derived a fuzzy linear programming problem. Lam and Wong [2] solved the fuzzy model of joint economic lot size problem with multiple price break by fuzzy non-linear programming problem. Recently, Roy and Maiti [3] solved the classical EOQ model in a fuzzy goal, fuzzy inventory costs and storage area by fuzzy non-linear programming method using different types of membership functions for inventory parameters. They [4] also developed the multi-objective inventory model under limited storage area and total average cost with stock dependent demand in fuzzy environment.

In this article multi-objective multi-item inventory model for deteriorating items with price dependent demand under limited storage area and total investment is developed in fuzzy environment. Here objectives are to maximize the profit and to minimize the wastage cost where profit goal, wastage cost, storage area and total investment are fuzzy in nature. In this model fuzzy parameters are represented by linear membership functions and after the fuzzification, it is solved by fuzzy programming technique. The model is illustrated numerically and results are obtained.

2. Assumptions:

i) Deterioration rate is age specific failure rate

ii) Demand Di is related to the unit price as:

$Di=\alpha i p i^{-\beta i}$ where $\alpha i>0$ and $\beta i (0<\beta i<1)$ are constants and real numbers.

3. Notations:

T : Scheduling time.

θi : Deterioration rate of ith item

Qi(t) : Inventory level at time t of ith item.

C_H : Total Holding cost.

C_{1i} : Holding cost per unit of ith item.

S_{di} : Total deteriorating units of ith item.

Pi : Selling price per unit of ith item.

Qi : Initial stock level of ith item.

(wavy bar (~) represents the fuzzification of the parameters)

4. Mathematical Formulation

4.1 Crisp Model

As qi(t) is the inventory level at time t of the i^{th} item, then

$$\frac{d}{dt}Q_i(t)+\theta Q_i(t)=-\alpha_i p_i^{-\beta_i} 0 \leq t \leq T_i$$

Using boundary conditions qi(t)=Qi at t=o and qi(t)=0 at t=Ti

$$T_i = \frac{1}{a_i}\log\left\{1+\frac{a_i P_i^{\beta_i} Q_i}{\alpha_i}\right\}$$

By neglecting a_i and higher powers of ai, Since a_i is very small positive number.

$$\therefore T_i = \frac{P_i{}^{\beta_i} Q_i}{\alpha_i}\left\{1-\frac{a_i P_i^{\beta_i} Q_i}{2\alpha_i}\right\}$$

The holding cost of i^{th} item in each cycle is C1iGi(Pi,Qi) Where,

$$G_i(P_i,Q_i) = \int_0^{Q_i} \frac{q_i dq_i}{\alpha_i P_i^{-\beta_i} + a_i q_i} = \frac{P_i^{\beta_i} Q_i^2}{2\alpha_i}\left\{1-2\frac{a_i P_i^{\beta_i} Q_i}{3\alpha_i}\right\}$$

The total number of deteriorating units of the i^{th} item is

$$\theta_i(Q_i)=a_i G_i(P_i,Q_i)$$

The net revenue for the i^{th} item is

$$N(Q_i) = (P_i-C_i)Q_i-P_i\theta_i(Q_i)$$

Hence the problem is

$$\text{Max PF}=\sum_{i=1}^{n}(N(Q_i)-C_{1i}G_i(P_i,Q_i)-C_{3i})$$

$$\text{Min WC}=\sum_{i=1}^{n} C_i\theta_i(Q_i)$$

Subject to;

$$\sum_{i=1}^{n} w_i Q_i \leq W$$

$$\sum_{i=1}^{n} C_i Q_i \leq B \qquad (*)$$

$$Q_i \geq 0, i=1,2,3,---,n$$

4.2 Fuzzy Model:

When the above profit goal, wastage cost, storage area and investment are fuzzy, the said crisp model becomes fuzzy model as :

$$\widetilde{M}\text{ax PF}=\sum_{i=1}^{n}(N(Q_i)-C_{1i}G_i(P_i,Q_i)-C_{3i})$$

$$\widetilde{M}\text{in WC}=\sum_{i=1}^{n} C_i\theta_i(Q_i)$$

Subject to;

$$\sum_{i=1}^{n} w_i Q_i \leq \widetilde{W} \qquad (**)$$

$$\sum_{i=1}^{n} C_i Q_i \leq \tilde{B}$$

$$Q_i \geq 0, i=1,2,3,---,n$$

5. Mathematical Analysis

5.1 Fuzzy Programming Technique

To solve the multi-objective programming problem by FPT, the first step is to assign two values Uk and Lk as upper and lower acceptable levels of achievement for the kth objective respectively and take dk=Uk-Lk=degradation allowance for kth objective (k=1,2). Now the problem defined in crisp environment is suitable for the application of FPT. The steps of the fuzzy programming technique is as follows:

Step1: Solve the multi-objective programming problem using only one objective at a time and ignoring the rest objectives subjected to the constraints of (*)

i) Problem with first objective PF(Qi) with constraints of (*)

$$\text{Max} \quad PF = \sum_{i=1}^{n} (N(Q_i) - C_{1i} G_i(P_i, Q_i) - C_{3i})$$

Subject to;

$$\sum_{i=1}^{n} w_i Q_i \leq W$$
$$\sum_{i=1}^{n} C_i Q_i \leq B$$
$$Q_i \geq 0, i=1,2,3,---,n$$

ii) Problem with first objective WC(Qi) with constraints of (*)

$$\text{Min} \quad WC = \sum_{i=1}^{n} C_i \theta_i (Q_i)$$

Subject to;

$$\sum_{i=1}^{n} w_i Q_i \leq W$$
$$\sum_{i=1}^{n} C_i Q_i \leq B$$
$$Q_i \geq 0, i=1,2,3,---,n$$

Let Q(1) and Q(2) be the optimum solution for the first and second objective functions.

Step 2: From the results of step 1, determine the corresponding values for every objective at each optimum solution derived. Using all the above optimum values of the objectives in step 1, construct a pay-off matrix (2x2) as follows :

$$\begin{array}{c} \\ Q1 \\ Q2 \end{array} \begin{array}{c} \begin{array}{cc} PF & WC \end{array} \\ \begin{bmatrix} PF(Q1) & WC(Q1) \\ PF(Q2) & WC(Q2) \end{bmatrix} \end{array}$$

Here, the values in the i^{th} row represent the optimum value of the i^{th} objective and the values of the other objective at Q(i). From the pay-off matrix, Lower bounds

L1= Min { PF(Q1) , PF(Q2) }

L2= Min { WC(Q1) , WC(Q2) }

and the upper bounds

U1= Max { PF(Q1) , PF(Q2) }

U2= Max { WC(Q1) , WC(Q2) }

are estimated as

$L_1 \leq \mathrm{PF}(Q) \leq U_1$ and $L_2 \leq \mathrm{WC}(Q) \leq U_2$

Step 3 : From step 2, we may find for each objective the values LK and UK corresponding to the set of solutions. For the multi-objective problem , the membership functions μ_{PF} which may be linear or non-linear are defined below. For simplicity, here we have considered linear membership functions only.

$$\mu_{\mathrm{PF}}(Q) = \begin{cases} 0 & ; \mathrm{PF}(Q) < \mathrm{L}_1 \\ \dfrac{\mathrm{PF}(Q) - \mathrm{L}_1}{U_1 - \mathrm{L}_1} ; & L_1 \leq \mathrm{PF}(Q) \leq U_1 \\ 1; \mathrm{PF}(Q) > \mathrm{U}_1 \end{cases}$$

$$\mu_{\mathrm{WC}}(Q) = \begin{cases} 0 & ; \mathrm{WC}(Q) > \mathrm{U}_2 \\ \dfrac{U_2 - \mathrm{WC}(Q)}{U_2 - \mathrm{L}_2} ; & L_2 \leq \mathrm{WC}(Q) \leq U_2 \\ 1; \mathrm{WC}(Q) < \mathrm{L}_2 \end{cases}$$

Step 4: find an equivalent crisp model by using a appropriate membership function for the initial fuzzy model.

If we use linear membership function defined as above then an equivalent crisp model for the fuzzy model can be formulated as:

Max α

Subject to;

$\mu_{\mathrm{PF}}(Q) \geq \alpha,$

$\mu_{\mathrm{WC}}(Q) \geq \alpha,$

$\sum_{i=1}^{n} W_i Q_i \leq \mathrm{W},$

$\sum_{i=1}^{n} C_i Q_i \leq \mathrm{B},$

$Q_i \geq 0, \alpha \in [0,1]$

6. Numerical Example

Input data : For n = 3 items, C1= 8, C2=8, C3=8, P1=12, P2=11, P3=10, C11=1.5, C12=0.5, C13=1.0, C31=100, C32=150, C33=120, α1=100, α2=90, α3=110, β1=0.08, a1=0.03, a2=0.028, a3=0.025, W1=0.5, W2=1.5, W3=0.3, W=2000sq.ft., B=2500.

Optimum Solution:

a) **Crisp Model :**

α=0.8235	Q1=128.1435	Q2=67.18442	Q3=117.1721
PF=279.7041	WC=45.076	W=200	B=2500

b) **Fuzzy Model:**

To illustrate the fuzzy model (**), we assume the above data with $\widetilde{PF}$= (280 , 290), $\widetilde{WC}$ = (45 , 50),$\widetilde{W}$= (200 , 220), $\tilde{B}$ =(2500 , 2600)

α=0.8501952	Q1=145.0718	Q2=66.39163	Q3=102.9091
PF=288.5019	WC=47.9479	W=202.9961	B=2514.9802

7. Concluding Remark

Until now, very few research work is available on multi-objective inventory models which have been solved by Fuzzy programming Technique (FPT). Here the multi-objective multi-item inventory model for deteriorating items is developed under limitations of storage area and total investment. The problem has been formulated in the fuzzy environment and solved by FPT algorithm. Corresponding optimum results are presented. Though the model considered here is with price-dependent demand, infinite replenishment and without shortages, the present analysis can be extended to other types of inventory models with finite replenishment, with shortages etc. The extension of this algorithm to other inventory models may be the area of future research

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